7. Gyakorlat:

$\boxed{6.F.}$ $f(x) := \ln(1+x)$ $(x > -1)$;

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$$

$$f''(x) = -1 \cdot (1+x)^{-2}$$

$$f'''(x) = 1 \cdot 2 \cdot (1+x)^{-3}$$

$$f^{IV}(x) = -1 \cdot 2 \cdot 3(1+x)^{-4}.$$

Sejtés:

$$f^{(n)}(x) = (-1)^{n+1} \cdot (n-1)! \cdot \frac{1}{(1+x)^n}, \quad n = 1,2,\ldots; \quad (x > -1)$$

Biz.: Tslj. ind. (HF)

$\boxed{5.F.}$ $h(x) = \sqrt{\frac{f(x)}{g(x)}}$

$f, g : I \to \mathbb{R}$, $f, g > 0$ $I$-n;

$f; g \in D(I) \Longrightarrow h \in D(I)$.

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7-8. Gyak:

$\boxed{1.F.}$ (a) $f(x) := 2 \cdot e^{x^2-4x}$ $(x \in \mathbb{R})$.

mon; lok. sz.é.

Meo. $f \in D$. | $f'(x) = 2 \cdot e^{x^2-4x} \cdot$

• $(2x-4) = 4e^{x^2-4x}(x-2)$.

<u>Mon.</u>: $f'(x) \gtrless 0 \iff x-2 \gtrless 0$

$\Rightarrow f'(x)>0$, ha $x>2 \Rightarrow (2,+\infty)$-en $f \uparrow$

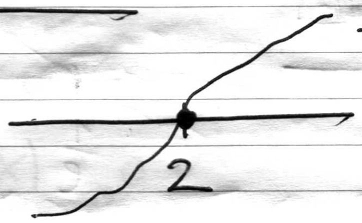
$f'(x)<0$, $x<2 \Rightarrow (-\infty,2)$-ön $f \downarrow$.

<u>Lok. szé.:</u>

Szüks. felt. $f'(x)=0 \iff \boxed{x=2}$

<u>itt lehet lok. szé.</u>

<u>Elégséges feltétel:</u>

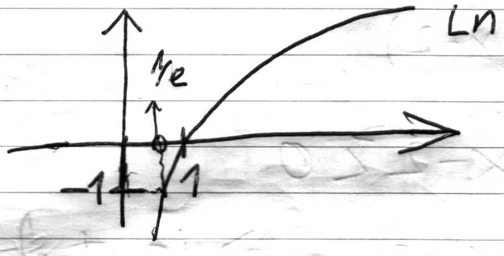



$\Rightarrow$ $\boxed{x=2 \text{ lok. min. hely.}}$

$\boxed{\text{1. F. b.}}$ $f(x) = x \cdot \ln x$ $(x>0)$.

Meo. $f \in D$; $f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1$ $(x>0)$.

Mon. $f'(x) \gtrless 0 \iff \ln x + 1 \gtrless 0 \iff \ln x \gtrless -1$.




$\ln x = -1 \iff x = e^{-1} = \frac{1}{e}$

Ha $0 < x < \frac{1}{e} \Rightarrow f'(x) < 0 \Rightarrow f \downarrow$ $(0, \frac{1}{e})$-n;

Ha $x > \frac{1}{e} \Rightarrow f'(x) > 0 \Rightarrow f \uparrow$ $(\frac{1}{e}, +\infty)$-en.

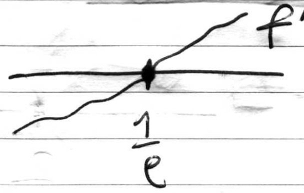
<u>Lok. sz. é.</u>

<u>Szüks. felt.:</u> $f'(x) = 0 \iff \ln x + 1 = 0$

$\iff \ln x = -1$

$\iff \boxed{x = \frac{1}{e}}$ itt lehet lok. sz. é.

<u>Elégséges felt.:</u>




$\Rightarrow \boxed{x = \frac{1}{e} \text{ lok. min. hely.}}$

$\boxed{2.F.}$ $f: \mathbb{R} \to \mathbb{R}$, $f \in D$, $f' = f$.

<u>Ig.:</u> $\exists \alpha \in \mathbb{R}: f(x) = \alpha \cdot e^x$ $(x \in \mathbb{R})$.

<u>Meo.:</u> ÖTLET: Tek. $\frac{f(x)}{e^x} \in D$;

$(f(x)/e^x)' = \frac{\overset{f(x)}{\overset{\|}{f'(x)}} e^x - f(x) \cdot e^x}{(e^x)^2} = 0$ $(\forall x \in \mathbb{R})$.

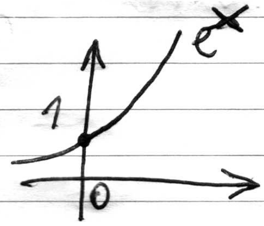
$\xRightarrow[\text{e.d.}]{\text{c.}}$ $\exists \alpha \in \mathbb{R}: \quad \dfrac{f(x)}{e^x} = \alpha \quad (\forall x \in \mathbb{R})$ ✓ $\blacksquare$

$\boxed{4.F.}$ (a). $\exists! \, x \in \mathbb{R}: \quad e^x = 1 + x$

<u>Meo.:</u> Bolzano + m.o.

$f(x) := 1 + x - e^x \quad (x \in \mathbb{R})$.

$f'(x) = 1 - e^x \quad (x \in \mathbb{R})$.




Ha $x > 0$, $f'(x) < 0$ $\Longrightarrow$ $f \downarrow$ $(0, +\infty)$-en.

Ha $x < 0$, $f'(x) > 0$ $\Longrightarrow$ $f \uparrow$ $(-\infty, 0)$-n.

$f(0) = 0 \Longleftrightarrow x_0 = 0$ meo. $e^x = 1 + x$.

$\forall x > 0: \; f(x) < 0$ ✓.

$\forall x < 0: \; f(x) < 0$ ✓

(b). <u>Biz. be.:</u> $x - \dfrac{x^2}{2} < \ln(x+1) < 0 \quad (\forall x > 0)$.

I, II

I. $\ln(x+1) < x \Longleftrightarrow 0 < \underbrace{x - \ln(1+x)}_{f(x)}$ $\forall x > 0$.

$f(0) = 0$; $\quad f'(x) = 1 - \dfrac{1}{1+x} = \dfrac{x}{1+x} > 0 \quad (\forall x > 0)$.

$\Rightarrow f\uparrow \quad (0,+\infty)$-en $\Rightarrow \boxed{I}$ igaz.

(b) $x - \frac{x^2}{2} < \ln(x+1) \quad (\forall x>0)$

$$\Updownarrow$$

$$0 < \underbrace{\ln(x+1) - x + \frac{x^2}{2}}_{f(x)} \quad (x>0).$$

$\underline{f(0)=0}$, $f'(x) = \frac{1}{x+1} - 1 + x = \frac{-x}{x+1} + x =$

$= x\left(1 - \frac{1}{x+1}\right) = x \cdot \frac{x}{x+1} > 0 \quad \forall x>0.$

$\Rightarrow \underline{f(x) > 0 \quad \forall x>0}$ ■

$\boxed{5.F.}$ $f:\mathbb{R}\to\mathbb{R}$, $f\in D$, $f(1)=10$;

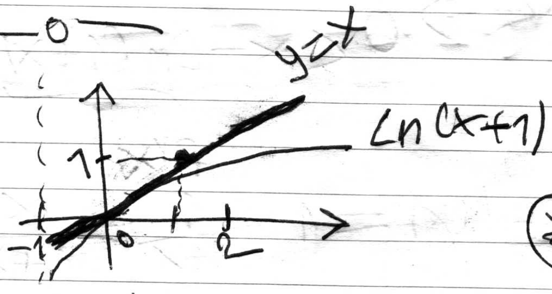
$f'(x)\in[1,4]$

$1\le x\le 4$

$? \le f(4) \le ?$

—o—

Mj.:



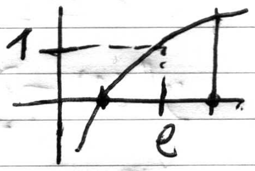

(*) $(\ln(x+1))'_{x=0}$.

⊛ $(\ln(x+1))'_{x=0} = \left(\frac{1}{1+x}\right)_{x=0} = 1$

$x - \frac{x^2}{2} = x\left(1 - \frac{x}{2}\right)$ ; $\ln 2 \geqq \frac{1}{2}$

$\iff 2\ln 2 = \ln 4 \geq 1$

$\ln 4 > 1$




Mj.:

[redacted] $\boxed{F.}$ $f: \mathbb{R} \to \mathbb{R}$, $f \in D$, $f(1) = 10$ ;

$f'(x) \in [1,4]$.

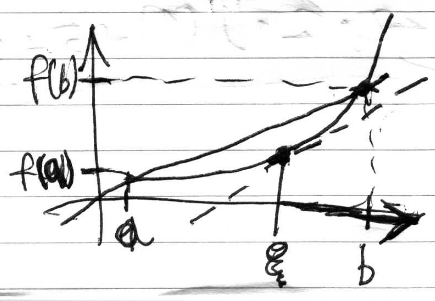
$x \in [1,4]$

? ... $\leq f(4) \leq$ ...

Meo.: Lagrange-k.é.t.

$$\left.\begin{array}{l} f \in C[a,b] \\ f \in D(a,b) \end{array}\right\} \implies \exists\, \xi \in (a,b): \frac{f(b)-f(a)}{b-a} = f'(\xi)$$

Mj.:




$f(1) = 10$; $[a,b] = [1,4]$

$\exists\, \xi \in [1,4]:$

$f(4) - f(1) = f'(\xi)(4-1)$

$1 \cdot (4-1) \leq f(4) - f(1) \leq 4 \cdot$

• $(4-1) \Longrightarrow 14 \leq f(4) \leq 26.$ $\blacksquare$

$\boxed{6.\ F.}$ $y \in \mathbb{R} \to \mathbb{R}$; ~~[illegible]~~ $y \in D^2$, $y(1)=1$

$$x^3 - 2x^2y^2(x) + 5x + y(x) - 5 = 0$$

$(\forall x \in D_y)$

$y''(1) = ?$

—o—

$y(1)=1$; $\quad x=1,\ y=1$

$$1^3 - 2\cdot 1^2 \cdot 1^2 + 5\cdot 1 + 1 - 5 \overset{?}{=} 0 \ ✓$$

$$3x^2 - 4xy^2(x) - 2x^2\cdot 2y(x)\cdot y'(x) + 5 + y'(x) = 0.$$

$$(*)\ \begin{cases} 6x - 4y^2(x) - 4x\cdot 2y(x)\cdot y'(x) - \\ - 4\big(2x\cdot y(x)\cdot y'(x) + x^2\cdot y'(x)\cdot y'(x) + x^2\cdot y(x)\cdot y''(x)\big) \\ + y''(x) = 0. \end{cases}$$

$\boxed{y''(1) = ?}$

$(*)$-ban: $x=1$, $y(1)=1$; $y'(1)$; $y''(1)$ -re

$$y'(1) \overset{\oplus}{\Longrightarrow} 3 - 4 - 2\cdot 2\cdot y'(1) + 5 + y'(1) = 0,$$

kifejezni